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GRAPHIC ARITHMETIC

BY

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GRAPHIC ARITHMETIC

By E. H. TAYLOR, B. S.
Instructor in Mathematics

In comparing numbers it is often convenient first to represent them by straight lines and then to compare the lines. As an illustration, take the values of the exports of the United States to four countries in 1901: (a) United Kingdom, $631,266,263; (b) Germany, $191,072,252; (c) Netherlands, $84,352,470; (d) France, $78,923,914. The relative size of these values can be more easily seen if they are represented by portions of a straight line.

The relation between two quantities that vary together may be shown by means of a line straight or curved, or a combination of lines. Such a representation is called a

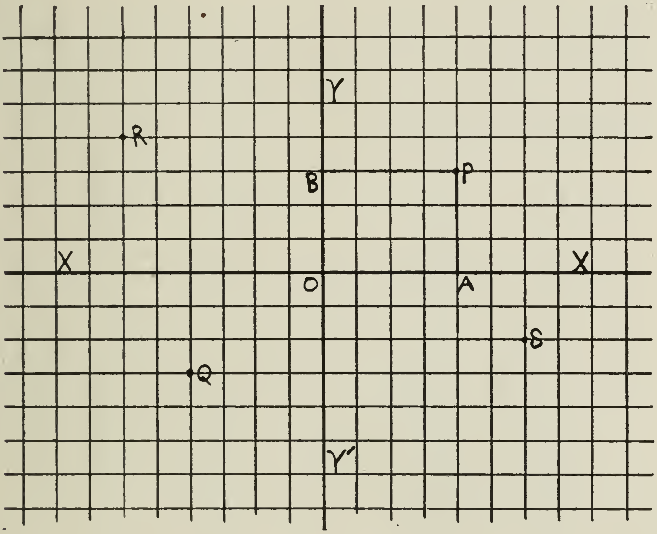
graph. The lines used may be considered as the path of a point in a plane, and any line may be located as nearly as we please by locating a sufficient number of its points. Therefore, to construct a graph, it is first necessary to learn how to locate a point in a plane.

If the streets of a city run toward the cardinal points, a point A on any street may be located with reference to a given point B, by giving the number of blocks north or south and the number east or west, from B to A. A point is located on the earth's surface by its latitude and longitude, the direction as well as the number of degrees of each being stated. In our government survey a township is located by its number north or south of a base line and east or west of a principal meridian. In each case—the place in the city, the point anywhere on the earth's surface, the township—the location is determined with reference to two lines intersecting at right angles by means of two measurements and their directions.

In the accompanying figure, let the two lines XX′ and YY′ intersect at right angles at O. From any point P in the plane of the two lines, draw PA and PB perpendicular to XX′ and YY′ respectively. OA the perpendicular distance of P from YY′ measured on XX′, is called its *abscissa*; OB the perpendicular distance of P from XX′, measured on YY′, is called its *ordinate*. The abscissa and ordinate of a point are called its *coördinates*. XX′ and YY′ are called the *axes of coördinates* and O is called the *origin*. XX′ is called the *axis of abscissas* or *X axis*; and YY′ the *axis of ordinates*, or *Y axis*. Abscissas are commonly represented by x, and ordinates by y. Thus, if one space be taken as the unit, for P we have $x=4$, $y=3$.

How shall we describe the position of Q? On a map we might say P is 3 units north and 4 units east of O; and Q is 3 units south and 4 west of O. But in mathematics opposite

directions are denoted by signs and we say that abscissas measured to the right of O are positive, to the left negative; ordinates measured upward from O are positive, downward, negative. We are now able to locate any point in the plane if we know its coördinates.



The coördinates of a point are written thus, (x, y), the abscissa being written first. Thus (4, 3) is the point P in the figure, the abscissa being 4, and the ordinate 3. The points P, Q, R, S are respectively (4, 3), (–4, –3), (–6, 4), (6, –2).

The student of arithmetic need not be frightened by the use of the negative quantity, even if this is his first experience with it, for in this discussion a negative quantity means simply one that is measured in the opposite direction from a positive quantity. As this is fundamental in the notion of

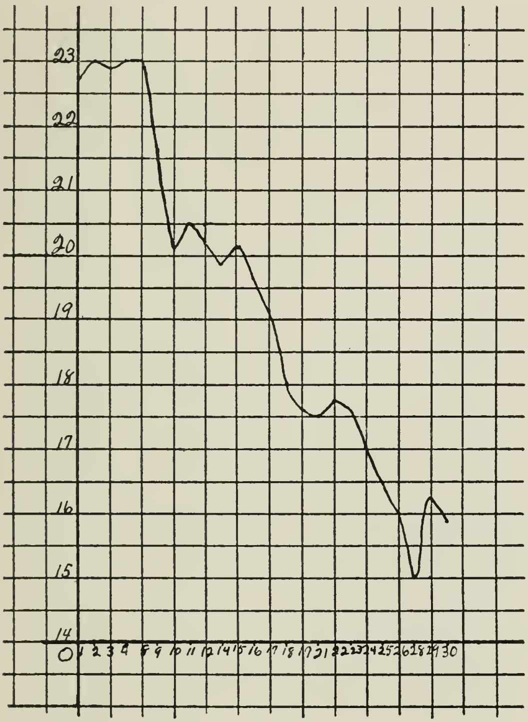
negative quantities in algebra, the discussion of the graph offers a good opportunity for an introduction to the study of this kind of quantity.

There are many useful applications of graphic methods to the interpretation of ordinary problems in arithmetic. For example, if we have the market price of an article through a given time, the variation may be seen quickly from the graph. The following are the closing bids for U. S. Steel stock in the New York market during September, 1903. There are no quotations, of course, for Sundays and none for Labor Day and the Saturday preceding.

Date	Price	Date	Price
Sept. 1	22¾	Sept. 17	19⅛
2	23	18	18
3	22⅞	19	17⅝
4	23	21	17½
8	23	22	17¾
9	21⅝	23	17⅝
10	20⅛	24	17
11	20½	25	16½
12	20¼	26	16
14	19⅞	28	15
15	20⅛	29	16¼
16	19⅝	30	15⅞

Let the time be measured from O to the right on the axis of abscissas, counting one half a space for each market day, beginning with Sept. 1. Beginning at O with 14 cents, let the prices be marked on the axis of ordinates above O, counting two spaces to a cent. If we wished to represent prices below 14 cents, they would be marked on the axis of ordinates below O. Likewise, time before Sept. 1, to the left of O on the axis of abscissas. Then the point that represents the price on Sept. 1 is on the axis of ordinates, 17½ spaces above

O; the one representing the price for Sept. 2, one half space to the right of the axis of ordinates and one half space higher than for Sept. 1; for Sept 3, one half space to the right and one-fourth space lower than for Sept. 2; and in the same way all



the points may be located. Now if these points are connected by a smooth curve or a series of straight lines, we have the desired graph. This graph shows more clearly than the mere prices, the general character of the market. If we wished to compare U. S. Steel with any other stock, this

could be best done by comparing the graphs. This comparison would probably suggest any relation that existed between them.

The graph may often be used advantageously in the discussion of the data involved in various economic questions. A glance at the accompanying figure will show the parallelism between, and the steady decline in, the freight rates by railroad and by lake and canal from 1870 to 1892 inclusive.

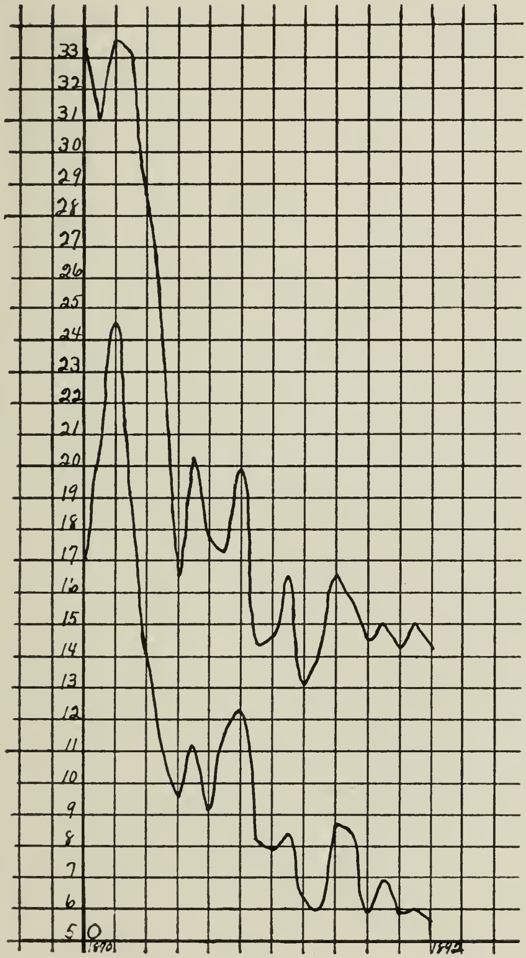
The following is the average freight charge on a bushel of wheat from Chicago to New York, all rail:

1870	33.3	1882	14.6
1871	31.0	1883	16.5
1872	33.5	1884	13.1
1873	33.2	1885	14.0
1874	28.7	1886	16.5
1875	24.1	1887	15.7
1876	16.5	1888	14.5
1877	20.3	1889	15.0
1878	17.7	1890	14.3
1879	17.3	1891	15.0
1880	19.9	1892	14.2
1881	14.4		

Average freight charge on a bushel of wheat from Chicago to New York, by lake and canal:

1870	17.1	1882	7.9
1871	20.2	1883	8.4
1872	24.5	1884	6.3
1873	19.2	1885	5.9
1874	14.1	1886	8.7
1875	11.4	1887	8.5
1876	9.6	1888	5.9
1877	11.2	1889	6.9
1878	9.2	1890	5.9
1879	11.6	1891	6.0
1880	12.3	1892	5.6
1881	8.2		

In the figure, the years are taken as abscissas, counting one half space to a year, beginning at O with 1870 and counting to the right; the prices are taken as ordinates, counting one space to a cent, beginning at O with five and counting upwards.



The difference beween simple and compound interest can by shown very clearly by a graph. Below is given the accumulated simple and compound interest at the end of each year for 9 years on $1 at 5 per cent:

Simple Interest		Compound Interest	
1	5	1	5
2	10	2	10.25
3	15	3	15.76
4	20	4	21.55
5	25	5	27.63
6	30	6	34.01
7	35	7	40.71
8	40	8	47.75
9	45	9	55.13

In the figure, the years are abscissas, counting two spaces to a year; the cents are ordinates, counting one half a space to a cent. The simple interest graph is a straight line, and the compound interest graph diverges from it, the divergence becoming more rapid as the time increases.

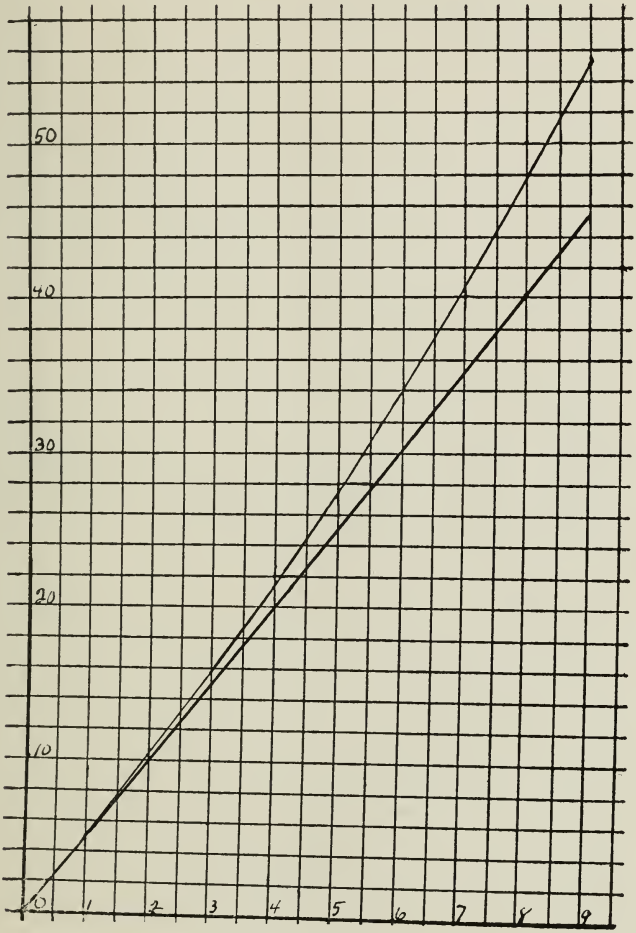
By plotting the interest graphs one may now solve graphicly such problems as the following: When will the compound interest on $1 at 5 per cent equal the simple interest at 6 per cent?

When will the amount of $500 drawing compound interest at 5 per cent, equal the amount of $600 at 5 per cent simple interest?

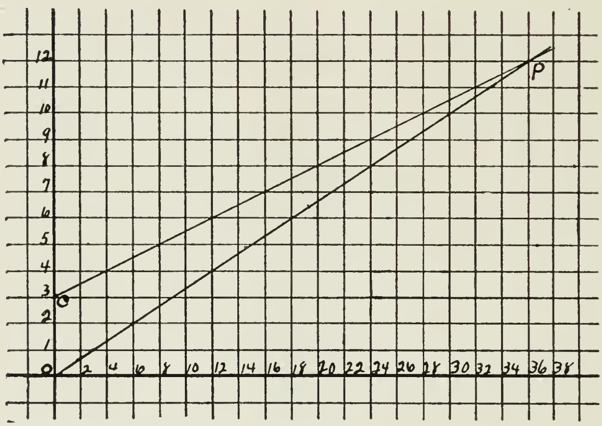
These problems may be solved graphicly by plotting the graphs, in the first case for the interests, in the second for the amounts, and finding the points of intersection. These solutions will be only approximate, of course; but any desired degree of accuracy may be secured by increasing the scale upon which the graphs are plotted.

Different methods of solution for the same problem are valuable, as they serve as checks. Such problems as the

following may be solved graphicly and the graphic solution is not only a check upon the numerical one, but gives it also a new meaning.



A starts from a given point and walks east at the rate of three miles an hour; three hours later B starts from the same point and walks in the same direction at the rate of four miles an hour. In how many hours will B overtake A and how many miles will A have traveled?

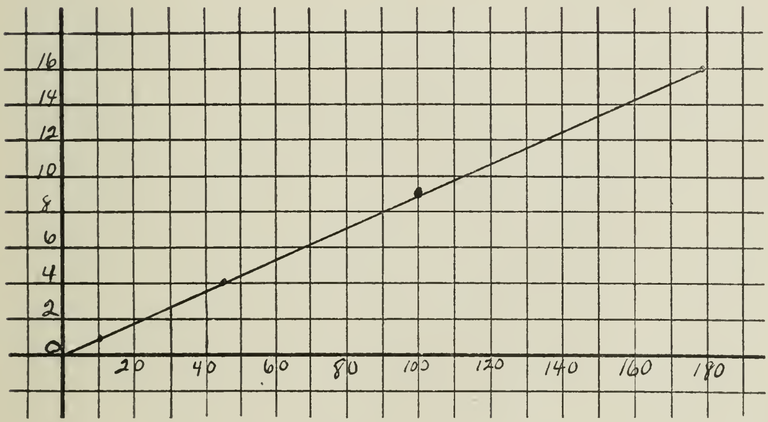


Take times for ordinates, and distances for abscissas. Let the time that A starts be at O, and count one space to an hour. Count one half a space to a mile. Finding now the points that correspond to A's location at the end of each hour and connecting them, we have the straight line OA. Since B starts from the same place three hours later, the point C corresponds to his position at starting. Finding the points that give B's location at the end of each hour and connecting them, we have the straight line CB. The point P where the two lines intersect corresponds to the point in the journey where the time and distance for A and B, as meas-

ured from the time A started and from the starting point respectively, are equal. The abscissa of P is 18 and its ordinatė 12, remembering that we counted one half space to a mile. Therefore A had traveled 12 hours and had gone 36 miles before he was overtaken. Since B started three hours later, he traveled nine hours. These are the same results, of course, as given by a numerical solution.

Nearly all teachers of physics use the graph in the discussion of the data obtained in the laboratory. With the aid of an Atwood's machine, the following values were obtained for the total space passed over by a falling body at the end of each second for the first four seconds:

1 sec.	11 cm.
2 "	45 "
3 "	100 "
4 "	178 "



By using the squares of the number of units of time as ordinates and the total spaces passed over as abscissas, we find the resulting graph is nearly a straight line, which shows

that the space increases uniformly with the square of the number of units of time; or to say it in another way, that the space passed over is proportional to the square of the number of units of time. If the times be used as ordinates and the total spaces as abscissas, the resulting curve indicates that the space passed over increases much faster than the time. Only the first graph, the straight line, is plotted here. In the figure, each unit in the squares corresponds to one half a spaces on the axis of ordinates and ten units of space passed over correspond to one space on the axis of abscissas.

The foregoing illustrations will indicate some of the kinds of problems that may be discussed graphicly. The list might be extended almost indefinitely and some more applications of this method will be suggested in the following discussion.

Much is being said and written about the failure to obtain results commensurate with the time used in the teaching of elementary mathematics. Some teachers seem to think the whole plan of instruction radically wrong. These usually have some pet scheme or other with which, if we follow directions, we may obtain the desired results. There is evident need of improvement in various ways and one place for this improvement is in the organization of the course of study. The efficiency of the teaching of elementary mathematics as a science, for its own sake, may be greatly increased by the proper correlation of arithmetic, algebra, and geometry in the primary and secondary schools. As taught in too many schools these subjects have almost no connection, and to pass from arithmetic to algebra or from algebra to geometry is like beginning the study of a new branch of knowledge. We should teach "less arithmetic, algebra, and geometry and more mathematics." This correlation means something more than a mere sandwiching together of some slices from the

different subjects. It involves a treatment that takes from all three whatever is useful in the solution of a given problem; uses the geometrical to supplement and illustrate the analytical treatment and simplifies and abbreviates geometrical proofs by using the methods and symbols of analysis; and leads easily and naturally from the particular discussions of arithmetic to the general ones of algebra. The general discussion of a problem if adapted to the advancment of the student, is of course more satisfactory than that of a particular case, and is often just as easy. Anything that tends to familiarize the student of arithmetic with the methods of the more advanced subjects is a step in the right direction, for the student can use these methods in arithmetical discussions and then knows something about them when he comes to their wider application. This may be given as one reason for studying the graph in arithmetic. The graph is used in the treatment of a number of topics in elementary algebra. Its applications there will mean much more if the student is already familiar with graphic solutions, for treating a new problem by a new method is not usually satisfactory. Besides, the solution of the arithmetical problems will suggest certain questions that will be further investigated in algebra; for instance, the meaning of the negative quantity and of the intersection of curves.

But this is not simply bringing a topic from algebra into arithmetic, for the graph is useful in the solution of problems in arithmetic. Some problems illustrating its use have already been given. Others can be found easily. If problems in arithmetic are taken from the data of observations made in elementary science while studying temperatures, barometric pressure, growth of plants, rainfall, and the like, much more meaning is given the data by discussing it with the aid of a graph. The data used for the graph will furnish material for problems in fractions and percentage. If the

numbers used are large, the construction of the curve gives practice in finding approximate relations between them. This is valuable to the student of arithmetic, for approximations, if seen quickly, may often be used as checks on long numerical computations. Plotting to scale is itself worth doing and is something that most students know nothing about.

The teaching of mathematics has improved lately as a result of the discussion of the correlation of mathematics with the other subjects of the curriculum, especially with physics. Teachers of physics say their students can not use the mathematics that they have studied and that the mathematics needed in the physics class must be taught there. There is no doubt some truth in this assertion,—how much need not concern us here. From the standpoint of physics and other sciences, the teaching of elementary mathematics may be much improved by teaching the parts of arithmetic, algebra, and geometry that are useful in those subjects, and by drawing the problems from their fields. For instance, for physics much attention should be paid to ratio, proportion, and variation, and instead of solving problems about hounds and hares, and pipes filling cisterns, the time could be better spent on problems involving the law of falling bodies, and Boyle's law. Surely the high school student's mathematics should be a useful tool, and to make it so he must have practice in applying it to the kind of problems that he will meet later. Those people who believe in practical results have some cause for complaint against the present teaching of mathematics. Some of the topics now studied in arithmetic and algebra classes should be displaced by others that have an equal disciplinary and greater practical value. It is thought that the subject of graphs is more interesting, and that it is more useful to the future student of physics and other sciences that receive mathematical treatment than much work now done in arithmetic. Therefore, it seems worth while to have the student of arithmetic do some work with graphs.

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The School Calendar.

1904.

SPRING TERM

March 29, Tuesday	*Class Work begins*
June 17, Friday	*Spring Term ends*

SUMMER TERM

June 20, Monday	*Classification*
June 21, Tuesday	*Class Work begins*
July 29, Friday	*Summer Term ends*